Exercise 5

Find an equation of the tangent line to the curve at the given point.

$$y = 4x - 3x^2, \quad (2, -4)$$

Solution

Start by finding the slope of the tangent line to the curve at $x = 2$.

$$\begin{aligned} m = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} &= \lim_{x \to 2} \frac{(4x - 3x^2) - [4(2) - 3(2)^2]}{x - 2} \\ &= \lim_{x \to 2} \frac{(4x - 3x^2) - (-4)}{x - 2} \\ &= \lim_{x \to 2} \frac{-3x^2 + 4x + 4}{x - 2} \\ &= -\lim_{x \to 2} \frac{3x^2 - 4x - 4}{x - 2} \\ &= -\lim_{x \to 2} \frac{(3x + 2)(x - 2)}{x - 2} \\ &= -\lim_{x \to 2} (3x + 2) \\ &= -[3(2) + 2] \\ &= -8 \end{aligned}$$

The general equation of a line is

$$y = mx + b.$$

Here the slope is $m = -8$.

$$y = -8x + b$$

Use the fact that the line passes through $(2, -4)$ to determine b.

$$-4 = -8(2) + b$$

$$-4 = -16 + b$$

$$b = 12$$

Therefore,

$$y = -8x + 12.$$

Below is a plot of the curve and the tangent line at $x = 2$.